Appendix 1

We present an optimization model for selecting a target to be used in conservation planning when the target-area function f is piecewise convex. Let $k = 1 \ldots K$ be indices for the targets and t_k be the numerical value of target k (e.g., 10%). Let a_k denote the area (in km^2) of a conservation area network selected with target k for each surrogate. Further, let b_k signify the slope of f to the left of target k: $b_k = \dfrac{a_k - a_{k-1}}{t_k - t_{k-1}}$. Note that if t_k is converted to units of km^2 then b_k is a dimensionless quantity that represents the effect of increasing the target from $k-1$ to k on the required size of the conservation area network. When f is relatively horizontal, the target can be increased without increasing the required area of the conservation area network substantially. Thus, one of the desiderata of the optimal target k^* is that the slope of f be as horizontal as possible to the left of k^*.

However, if f consists of more than one relatively horizontal line segment, it is desirable for conservation purposes to select the highest target that corresponds to a relatively horizontal segment of f. The Introduction explains why higher targets are desirable from a conservation standpoint.

We include the following constraints in the optimization model to ensure that the model selects as high a conservation target as possible. Let $\bar{l}_k$ denote the average slope of the target-area function for targets less than k, i.e. $\bar{l}_k = \dfrac{1}{k-1}\sum_{j=1}^{k-1} b_j$, and let $\bar{u}_k$

symbolize the average slope for targets greater than k, i.e. $\bar{u}_k = \frac{1}{K-k}\sum_{j=k+1}^{K} b_j$. When the target level k is small, $\bar{l}_k$ tends to be greater than $\bar{u}_k$. Our optimization model requires that the selected target level k^* must have the property that $\bar{l}_{k^*} \le b_{k^*} \le \bar{u}_{k^*}$. The model also requires that b_{k^*} be as horizontal as possible. If f has more than one relatively horizontal line segment, these requirements pick the highest target associated with a relatively horizontal segment. For example, the target-area function of the Queensland plant data set (Figure 2) is relatively horizontal for the 10% target ($b = 4.81$), the 35% target ($b = 25.26$), and the 60% target ($b = 24$); however, only for the 60% target is b at least as large as the average slope for smaller targets ($\bar{l} = 23.47$) and no greater than the average slope for larger targets ($\bar{u} = 66.53$). We formulate the optimization model as follows:

minimize $$\sum_{k=1}^{K} w_k \quad (1)$$

subject to $$w_k \ge b_k x_k - 1, \quad k = 1,\ldots,K \quad (2)$$

$$\sum_{k=1}^{K} x_k = 1 \quad (3)$$

$$b_k x_k \le \bar{u}_k + M y_k, \quad k = 1,\ldots,K \quad (4)$$

$$-b_k x_k \le M y_k - \bar{l}_k, \quad k = 1,\ldots,K \quad (5)$$

$$x_k \le M(1 - y_k), \quad k = 1,\ldots,K \quad (6)$$

$$x_k \in \{0,1\}, \quad k = 1,\ldots,K \quad (7)$$

$$y_k \in \{0,1\}, \quad k = 1,\ldots,K \quad (8)$$

$$w_k \geq 0, \qquad k = 1, \ldots, K \quad (9)$$

For each target k, constraint (2) computes the difference between one and the slope of the target-area function f associated with k (recall that a function with a slope of one is horizontal). Together with constraint (2), the non-negativity constraint (9) captures $\max\{0, b_k x_k - 1\}$, where $b_k x_k - 1$ is the difference between the slope of a horizontal line and the slope of the target-area function to the left of target k. The objective of the model (constraint 1) is to pick a target k such that this difference is as small as possible. Constraint (3) requires that exactly one target level be selected for use in the conservation plan. x_k is a binary decision variable set equal to one if target k is selected for use in the conservation plan and set equal to zero otherwise. Constraints (4)-(6) require that for each target level k either $\bar{l}_k \leq b_k \leq \bar{u}_k$ or $x_k \leq 0$. That is, we can only select a target k if the slope of f to the left of k is between the average slope for targets less than k and the average slope for targets greater than k. M is a large positive constant and y_k is a dummy variable; both are used to formulate either-or constraints (Hillier & Lieberman 2001).

Hillier, F. S. and G. J. Lieberman. 2001. Introduction to Operations Research. Seventh Edition. McGraw-Hill, Boston.