Abstract

In-situ permanent sensors allow the possibility of monitoring in real-time dynamic changes in formation properties due to primary or enhanced hydrocarbon recovery. A feedback loop can be enforced in response to in-situ permanent sensor data to optimally control the recovery of hydrocarbon assets. Prototypes of in-situ permanent sensors include pressure gauges, acoustic geophones, and direct-current (DC) resistivity electrodes.

In the past, quantitative studies have been performed to assess the value of in-situ permanent pressure data to mapping permeability variations in close proximity to the sensor deployment. Malinverno and Torres-Verdín on the other hand, have described a procedure to estimate water invasion profiles using in-situ DC resistivity sensors. However, to date little is known about the possibility of estimating petrophysical variables that jointly honor pressure and DC resistivity data acquired with in-situ permanent sensors. The two types of data sense different and complementary components of the underlying petrophysical model, and it seems appropriate to develop a quantitative estimation procedure that can assimilate the best elements of the two types of measurements. In this paper, we develop a methodology to jointly invert pressure and DC resistivity data acquired with in-situ permanent sensors in a hypothetical water injection experiment. Time-variable flow rates are enforced while injecting water into the surrounding rock formations, hence producing a sequence of repeated pressure pulses. The objective of the experiment is to estimate permeabilities, porosities, and fluid distributions that honor (a) sensor data acquired during a single pressure pulse, and (b) a more complete time record of pressure data acquired starting from the early stages of water injection. Our inversion results shed new light to the independent and joint sensitivity of noisy pressure and DC resistivity data to detecting dynamic perturbations of petrophysical properties. Examples of inversion are used to assess the relative benefits of different types of sensor deployments.

Introduction

Inversion of pressure transient data into spatial distributions of permeability has traditionally been a central element of the characterization and evaluation of hydrocarbon reservoirs. Data for this type of inversion consists of time records of fluid production and pressure. Examples of nonlinear inversion problems arising in the context of reservoir characterization can be found in Chen et al., Chavent et al., He et al., Wu et al., Wu, and Wu and Datta-Gupta. More recently, the availability of permanently installed downhole sensors has created a renewed interest in the solution of complex inverse problems in hydrocarbon reservoir characterization. A continuous space and time-domain data-stream is now available to conduct real-time monitoring of the variation of fluid flow parameters resulting from production (or injection) schedules. Prototypes of permanent sensors have also been constructed for behind-casing deployment. When commercially available, in-situ sensors will offer the means of delivering real-time images of the spatial distribution of petrophysical properties not only in the proximity of a well but also between existing wells. These sensors will eventually serve as tools for real-time formation evaluation as well as for reactive reservoir management. The petrophysical information inferred through the interpretation of in-situ sensor data could be utilized in applications such as opening or closing of production intervals in response to the breakthrough of unwanted fluids, or in the early detection of an advancing water front prior to breakthrough. Permanently deployed resistivity arrays have been successfully tested to monitor fluid movement at the near and far-field reservoir scale.

The problem considered in this paper consists of mapping discrete time- and space-domain variations of transient pressure data as well as discrete space-domain measurements of direct-current (DC) resistivity data, into spatial distribution...
of petrophysical properties. We focus our work to the case of axisymmetric spatial models of permeability, porosity, and electrical resistivity. The specific geometrical model considered in this paper is illustrated in Fig. 1. A nonlinear optimization algorithm is developed to jointly invert pressure and electric potential data acquired with in-situ permanent sensors in a hypothetical water injection experiment. The objective of the experiment is to estimate axisymmetric spatial distributions of permeability, porosity, and electrical resistivity, that honor (a) sensor data acquired during a single shut-in period subsequent to a constant step-injection rate pulse, and (b) a more complete time record of data acquired starting from the early stages of water injection experiment. We also investigate the sensitivity of the inversion results to the impact of random measurement noise.

**Modeling Overview**

Quantitative studies have been performed to assess the value of in-situ pressure permanent sensor data to detecting permeability variations in the proximity of a sensor deployment. In this paper, components of the petrophysical model under investigation consist of single-phase permeability, porosity, and electrical resistivity. We assume that the unknown petrophysical model can be parameterized with a finite and discrete number of values. The corresponding inverse problem is solved by minimizing a quadratic cost function written as the sum of the square differences between the measured data and the data yielded by a forward modeling algorithm. A nonlinear Gauss-Newton fixed-point iteration search is used to minimize the quadratic cost function. This minimization strategy requires the computation of the Jacobian (sensitivity) matrix whose entries consist of first-order variations of the cost function. Construction of the Jacobian matrix is the most computationally demanding component of the nonlinear inversion algorithm. In the past, extensive research has been undertaken to approximate, eliminate, and economize the computation of the Jacobian matrix. Torres-Verdin and Habashy and Ellis et al. present alternative approaches for the effective computation of the Jacobian matrix. The inversion algorithm developed in this paper is based on an efficient least-squares minimization technique adapted from the work of Torres-Verdin et al. We also make use of a novel dual-grid approach to accelerate the inversion. This latter approach renders our optimization algorithm efficient for the large-scale parametric inversion required in practical reservoir applications.

**Numerical simulation of single-phase fluid-flow phenomena.** For the modeling of fluid flow, we use an algorithm that solves the governing partial differential equation on a 2D axisymmetric finite-difference spatial grid. The original diffusivity equation is converted into an equivalent finite-difference operator problem that is solved with an Extended Krylov Subspace Method (EKSM). This simulation strategy yields multiple-time pressure results with almost the same computer efficiency as that of a single-time simulation. A brief description of the single-phase fluid flow numerical algorithm is presented in Appendix A.

Both accuracy and efficiency of the EKSM were quantified extensively by means of benchmark comparisons against the commercial reservoir simulator ECLIPSE 100. The numerical simulations performed with an EKSM-based computer code (EKSM) agree within 1% of ECLIPSE 100 simulations. We show an example case from our validation study in Figs. 2(a), 2(b), and 2(c). Water is injected into the multiblock reservoir shown in Fig. 2(a) during 135hr at a constant rate of 500rbl/d. The pressure fall-off response resulting from a 100hr-long shut-in period was simulated using EKSM and ECLIPSE 100. Figures 2(b) and 2(c) show a comparison of numerical simulations of pressure computed for two spatial locations within the reservoir. For both of these locations, EKSM and ECLIPSE 100 simulations agree within 1% of each other.

**Inverse Modeling of Multiphysics Data via Nonlinear Optimization**

The inversion approach developed in this paper is based on the cooperative use of electrical and fluid-flow measurements acquired in porous and permeable rocks. Injection of water into otherwise hydrocarbon saturated rocks causes a variation of electrical resistivity due to the contrast in electrical resistivity between in-situ and injected fluids. When water is injected through a borehole, the variation of resistivity becomes space dependent within the surrounding rock formation. For a homogeneous rock formation, the distribution of injected fluids around the borehole becomes symmetric with respect to the axis of the borehole. Under some restrictive assumptions, the radial distribution of fluids away from the injection well can be approximated by sharp boundaries between coaxial-cylindrical blocks. Each saturation block can be described by an equal-size resistivity block due to an explicit relationship between saturation and resistivity, i.e., Archie’s law. In addition, an approximate simulation of multi-phase flow can be performed by describing the dependence of effective permeabilities on saturation via...
discrete permeability segments rather than by making use of explicit relative permeability relationships. In this paper, we assume a negligible salinity gradient between injected and in-situ irreducible water. Consequently, for a water injection application in an axisymmetric single-well geometry, the permeability segments used to approximate multi-phase flow naturally coincide with the coaxial-cylindrical resistivity blocks that describe a variation in fluid saturation.

From a measurement viewpoint, the physical signature of the resistivity distribution can be captured by electrical measurements (i.e. voltages) resulting from a DC source excitation. On the other hand, the spatial distributions of permeability and porosity govern the pressure response of the medium surrounding the injection well. Although spatial variations of electrical resistivity and permeability can cause uncoupled perturbations on pressure and electric voltage measurements, an indirect coupling exists between the description of the physics of fluid flow phenomena and the physics of DC electrical phenomena. In effect, the parameter, \( r \), that defines the radial location of each petrophysical block is common to the description and numerical simulation of the two types of data. We assume that information about the vertical distribution of petrophysical blocks, i.e., the thickness of each layer, is readily available from other types of borehole measurements (e.g. well logs). Thus, in the simulation of electrical and pressure data, we set \( r_h \) equal to \( r_k \) and thereafter jointly invert the two types of measurements to yield values of electrical resistivity and permeability within each petrophysical block. We remark, however, that in general a strong link exists between the distributions of electrical resistivity, permeability, and porosity in the general context of multi-phase fluid flow in porous media. Empirical formulas that relate permeability and porosity have been reported in the literature (see, for instance, Dussan V. et al.23). High permeability formations tend to be associated with relatively high porosities and vice versa. Based on the latter remark, in this paper we additionally explore the joint inversion of voltage and pressure data for the estimation of spatial distributions of electrical resistivity and porosity. Analogous to the case of permeability and resistivity, we link both parameters through the radial location of block boundaries.

The motivation in developing a joint inversion approach for pressure and DC resistivity data is to estimate petrophysical models that are consistent with the two types of measurements. Even though the two measurements are different in nature, they remain sensitive to a common subset of the global set of unknown petrophysical parameters. The existence of a common subset of model parameters provides an efficient way to reduce non-uniqueness in the inversions otherwise performed independently with the two sets of measurements.

For the estimation of the 2D axisymmetric spatial distribution of petrophysical parameters, we choose to partition the permeable medium into a set of non-overlapping spatial segments. Permeability (or porosity) and electrical resistivity are assigned a constant value within each spatial segment. Let \( \mathbf{m} \) be the size-\( N \) vector of unknown parameters that fully describe the axisymmetric distribution of petrophysical parameters, and \( \mathbf{m}_k \) a reference vector of the same size as \( \mathbf{m} \) that has been determined from some a priori information. We undertake the estimation (inversion) of \( \mathbf{m} \) from the measured data by minimizing a quadratic cost function, \( C(\mathbf{m}) \), defined as

\[
2C(\mathbf{m}) = \| W_d \left( \mathbf{d}(\mathbf{m}) - \mathbf{d}^{\text{obs}} \right) \|^2 + \lambda \| W_m : (\mathbf{m} - \mathbf{m}_k) \|^2,
\]

(1)

where \( \mathbf{d}^{\text{obs}} \) is a size-\( N \) vector that contains the noisy measured data, and \( W_d \) is the inverse of the data covariance matrix. This data-weighing matrix describes the estimated variance for each particular data sample and the estimated correlation between data sample errors10. \( \chi^2 \) is the prescribed value of enforced data misfit. A priori estimates of the noise in the data are employed to determine the magnitude of \( \chi^2 \). \( \mathbf{d}(\mathbf{m}) \) is the data vector numerically simulated for specific values of \( \mathbf{m} \). \( W_m \) is the inverse of the model covariance matrix, used to enforce a quantitative degree of confidence in the reference model, \( \mathbf{m}_k \), and to provide a priori information. \( \lambda \) is a Lagrange multiplier or regularization parameter. In this paper, we make use of an efficient strategy for the computation of the regularization parameter described in Torres-Verdín and Habashy10.

The first additive term on the right-hand side of Eq. (1) drives the inversion toward fitting the data within the desired \( \chi^2 \) value. The sole presence of such a term in the cost function, \( C(\mathbf{m}) \), will yield multi-valued solutions of the inverse problem as a result of both noisy measurements as well as insufficient and imperfect data sampling. Enforcing an extremely small data misfit may result in petrophysical models with exceedingly large model norms10. The second additive term on the right-hand-side of Eq. (1) is used to reduce non-uniqueness and to stabilize the inversion in the presence of noisy and sparse data. In this context, the Lagrange multiplier, \( \lambda \), controls the relative weight of the two additive terms in the cost function24.

**Data and model vectors.** In the above cost function, the data vector \( \mathbf{d}^{\text{obs}} \) is constructed from discrete real values of DC resistivity data gathered in the form of electric potentials (voltages or voltage differences), \( u \), and/or pressure, \( p \), in the following organized fashion:

\[
\mathbf{d}^{\text{obs}} = [u_1, u_2, \ldots, u_j, p_{i+1}, \ldots, p_M]^T,
\]

(2)

where \( M \) is the number of measurements and the superscript \( T \) indicates transpose. The types of measurements to be included in the data vector will depend on the inversion
approach. For the option of an independent inversion, only either pressure or electric potential will be included in the measurement vector. An inversion performed with a single data type will be intended to yield only estimates of its associated petrophysical parameters. The joint inversion approach will make use of both types of measurements, and will be intended to produce estimates of all of the common and uncommon model parameters.

The ordering procedure that assigns an index, \( j \), to a given measurement is a function of measurement depth for the DC electrodes, sensor (receiver) locations, and measurement time (for pressure data). Similarly, the model vector can be assembled as follows:

\[
m = [R_1, R_2, \ldots, R_l, r_{i+1}, \ldots, r_m, k_{m+1}, \ldots, k_n]^T
\]

for the option of joint resistivity-permeability inversion, and

\[
m = [R_1, R_2, \ldots, R_l, r_{i+1}, \ldots, r_m, \phi_{n+1}, \ldots, \phi_n]^T
\]

for the option of a joint resistivity-porosity inversion. By denoting the model parameters as \( m_i = 1, 2, \ldots, N \), we have

\[
m = [m_1, m_2, \ldots, m_N]^T,
\]

where \( R \) represents electrical resistivity, \( k \), permeability, and \( \phi \), porosity. The model parameter \( r \) stands for radial location of a block boundary. For the water injection experiment, \( r \) describes the radius of the saturation front separating the oil and water banks.

A model vector constructed with only the model parameters \( R \) and \( r = r_R \) is used for the independent inversion of electrical voltages. Likewise, an independent inversion of pressure data will involve only the model parameters \( k \) (or \( \phi \)), and \( r = r_k \). For both joint and independent inversion approaches, an arbitrary combination of individual model parameters can be estimated depending on the preliminary information available for the unknown model.

Given that all of our model parameters are real and positive, we implement the change of variable \( \mu_j = \ln(m_j) \) except for porosity, \( \phi_j \). This change of variable is consistent with the fact that both electrical resistivity and permeability usually exhibit a large degree of variability. Previous experience has proven that both electrical resistivity (or conductivity) and permeability are better represented on a logarithmic scale\(^{[6, 11]}\). However, the same statement does not hold true for porosity. Porosity is defined to be a pore volume fraction and, as such, usually does not span a wide range of values.

**Gauss-Newton fixed point iteration search.** To determine a stationary point, \( m \), where the cost function attains a minimum, we use a Gauss-Newton fixed-point iteration search\(^{[3]}\). This method considers only first-order variations of the cost function in the neighborhood of a local iteration point. The corresponding iterated formula can be written as

\[
m^{k+1} = [J^T(m^k) \cdot W^T_d \cdot W_d \cdot J(m^k) + \lambda W^T_m \cdot W_m]^{-1} \cdot [J^T(m^k) \cdot W^T_d \cdot W_d \cdot [d(m^k) - d_{\text{obs}} + J(m^k) \cdot m^k] + \lambda W^T_m \cdot W_m \cdot m_R],
\]

subject to

\[
l_i \leq m_i^{k+1} \leq u_j.
\]

In the above expression, the superscript \( k \) is used as an iteration count, and \( J(m) \) is the Jacobian matrix of \( C(m) \).

Upper and lower bounds enforced on \( m^{k+1} \) are intended to have the iterated solution yield only physically consistent results.

The fixed-point iteration search for a minimum of \( C(m) \) is concluded when the measured data have been fit within the prescribed tolerance, \( \chi^2 \). A derivation of the iterated formula given in Eq. (5) is presented in Appendix C.

The inversion algorithm employed in this paper also takes advantage of a novel cascade optimization technique that incorporates a dual finite-difference gridding approach to accelerate the inversion associated with a large number of unknown model parameters. This highly efficient least-squares minimization technique is adapted from the work of Torres-Verdín et al.\(^{[12]}\).

**Sensitivity Studies for the Joint Inversion of Permanent Sensor Data**

The foregoing nonlinear inversion algorithm is applied to the estimation of petrophysical model parameters, namely, permeabilities, porosities, and electrical resistivities assuming a 2D axisymmetric geometry. Our objective is to advance a proof of concept for the cooperative use of pressure and DC resistivity data.

**Six-block test case.** The actual petrophysical model is described in Fig. 3(a). This figure shows a vertical cross-section of the axisymmetric permeability, porosity, and resistivity field around a vertical injection well. A vertical observation well is assumed to be drilled 72.2m away from the injection well. Both the injection and the observation wells are equipped with 15 in-situ permanent pressure sensors; 11 of which are installed across the formation of interest and are evenly distributed along the corresponding well. In addition, 2 sensor couples are deployed at the top and bottom of the reservoir within the sealing non-permeable layers.

From the viewpoint of electrical hardware, we assume a uniformly distributed DC resistivity array deployed along the injection well. The resistivity array consists of 18 point-contact electrodes. Reservoir and fluid flow parameters associated with this inversion exercise are listed in Table 1. Input data for inversion were simulated numerically using the forward modeling codes EKSMS and NKARD.
A finite difference grid of size $134 \times 249$, radial and vertical nodes, respectively, was constructed to perform the numerical simulations and inversions of pressure. This grid, shown in Fig. 3(b), consists of logarithmic steps in the radial direction, and a combination of logarithmic and linear steps in the vertical direction. Figure 3(c) displays a plot of the simulated pressure response, $\Delta p$ vs. $t$, involving pressure data from all of the sensors deployed along the injection and observation wells. The associated time record of flow-rate measurements, $q$ vs. $t$, is superimposed to the plot of simulated pressure. Figures 3(d) and 3(e) also show simulated pressure data for sensors deployed along injection and observation wells separately as a function of sensor location and time. Due to cross-flow among reservoir blocks, the sensitivity of pressure data to time variations of injection flow rate is relatively smaller along the observation well than along the injection well.

For the simulation of the DC electrical data, we constructed a 201-node logarithmically distributed radial mesh to be used with NKARD. The spatial distribution of nodes for this grid is shown in Fig. 4(a). Upper and lower reservoir boundaries are displayed together with the radial nodes. Fig. 4(b) shows the electric potential data simulated along the electrode arrays as a function of both sensor number and measurement depth. Single-time electric voltages measured at the end of the water injection schedule constitute the electrical data input to the inversion algorithm.

In this paper, we investigate the inversion of (a) pressure data acquired at one single-pulse schedule, namely, first fall-off period from 100 to 150hr, and (b) a more complete time record of data acquired starting from the early stages of water injection experiment (i.e. within the time interval from 100 to 300hr shown in Fig. 3(c)). Henceforth, we refer to the latter data set as single-pulse and to the former type of data set as multi-pulse pressure data.

Inversions with noise-free data. We first perform independent inversions for permeability, porosity, and resistivity fields. Only pressure data are used for the inversion of permeabilities and porosities, whereas, only electric potential data are used for the inversion of resistivities. Next, we conduct joint inversions of permeability-resistivity, and porosity-resistivity. Six-block model shown in Fig. 3(a) is used as a reference for the inversion results. Figure 5 shows vertical cross-sections (radial distance vs. vertical location) of the actual and post-inversion permeability fields obtained with both independent and joint inversion approaches. Inversion results obtained with the usage of single- and multi-pulse pressure data for both inversion approaches are also shown in Fig. 5. The spatial location of in-situ pressure sensors deployed along the injection and observation wells is displayed on the same cross-section plots. Analogous sets of plots for porosity and electrical resistivity fields are shown in Figs. 6 and 7. The spatial location of the 18-electrode resistivity array is superimposed to the cross-section of resistivity shown in Fig. 7. For the inversion of model parameters $k$, $\phi$, and $r$, the mean value of each individual parameter is used as the initial guess for the inversions. However, for $R$, the initial guess is constructed from the mean value of each vertical set of segments.

In all cases, post-inversion model-domain errors are rendered better than 1%. Although model domain percent errors are significantly small for the entire exercise, results obtained with the joint inversion approach consistently yield smaller model-domain percent errors than those yielded by independent inversions. For instance, for the independent inversion of permeability field with multi-pulse pressure data, the maximum model domain error is 0.294%, whereas, for an equivalent joint permeability-resistivity inversion, the maximum model domain error drops to 0.102%. On the other hand, comparison of the inversion results from the viewpoint of single- versus multi-pulse pressure data did not indicate a significant advantage of one approach over the other for the investigated noise-free cases.

We also computed the relative data misfit using the formula

$$
\frac{\| W_d \cdot (d(m^k) - d^{obs}) \|^2}{\| W_d \cdot d^{obs} \|^2}.
$$

(7)

Relative data misfit values were better than $1.0 \times 10^{-4}$ for all of the noise-free inversions presented in this section. An example of post-inversion data domain fit for the injection well pressures is shown in Fig. 8 for the case of joint porosity-resistivity inversion. Excellent data fits were common for inversions performed with noise-free data.

Inversion with noisy data. We also assessed the influence of noisy measurements on the inverted six-block model parameters. In this paper, we consider a test example where the simulated pressure data were contaminated with 1% zero-mean random Gaussian noise. To generate noise, we make use of a random number generator of standard deviation equal to the given percentage of amplitude of pressure change. Figures 9(b) and (c) display the set of noisy single- and multi-pulse pressure data inversions of porosity, respectively. The actual porosity field is also shown in Fig. 9(a). Analogous inversion results of noisy single- and multi-pulse pressure data for the permeability field are shown in Figs. 10(b) and (c). Again, Fig. 10(a) displays the actual permeability field for comparison purposes. Inversion results of noisy measurements indicate that the suggested inversion approach allows the robust estimation of petrophysical parameters under the deleterious influence of noise. Model parameters for segments closer to the injection well are estimated more accurately than for segments located farther away from the observation well. Comparisons of Figs. 9(b) and 9(c) to Fig. 9(a) show that the usage of multi-pulse pressure data yields more accurate estimates of porosity compared to those inverted from single-pulse data. A similar comparison is also presented for the noisy permeability field inversion shown in Figs. 10(a), 10(b),...
and 10(c). For this case, inverted permeability values are closer to the actual model when inverted with multi-pulse pressure data. However, the adverse effect of noise becomes evident in the horizontal boundary location inverted for the third layer from the top.

For the inversion with noisy data, stability was achieved by setting the matrix $W_m$ equal to a unity diagonal matrix in Eq. (1). The Lagrange multiplier, $\lambda$, in Eq. (1) then takes the role of a Wiener regularization constant. We implemented an adaptive Lagrange multiplier search technique such that at the initial steps of the iteration, a misfit reduction of at least 50% was enforced at each iteration with respect to the data misfit achieved at the previous iteration. This approach assigns large $\lambda$ values to Eq. (5) when $\Delta(m^k) = m^{k+1} - m^k$ undergoes large variations in the initial steps of the iteration process. In effect, more weight is allocated to the second term of the cost function in Eq. (1) when $\Delta(m^k)$ undergoes large changes. This renders the search direction into steepest descent, which is a more appropriate approach to use at the initial steps of the iteration process. On the other hand, when $\Delta(m^k)$ undergoes small variations, $\lambda$ takes small values allocating more weight to the first term of the cost function. The search direction corresponds to Gauss-Newton direction, which is a more appropriate approach closer to the minimum.

Application of a dual-grid technique in conjunction with the joint inversion approach. By developing a methodology for joint inversion, we attempt to take advantage of the pool of information supplied by measurements from multiple types of permanent sensors. On the other hand, this leads the inversion algorithm to deal with extra computational loads. One way to overcome this problem is using non-conventional, yet, fast and robust inversion schemes. In this paper, we implement a dual-grid inversion approach using the cascade minimization technique introduced by Torres-Verdín et al. In this algorithm, an inner loop approximates the Jacobian matrix with fast coarse-grid simulations while an outer loop controls the direction of convergence through updates of fine-grid numerical simulations. Inner and outer loops can be designed in a flexible fashion to improve convergence and reduce computation times. Computational performance of coarse-grid inner loop calculations can be further improved by means of the Broyden’s rank-one update formula for the Jacobian matrix (see Appendix C). Torres-Verdín et al. present the necessary convergence condition to be satisfied by the dual-grid inversion procedure. Table 2 shows a comparison of CPU times (clocked on a SGI Octane 300MHz machine) required by the inversions performed with various minimization strategies for a six-block test case. The formation model considered for this inversion exercise is a slightly larger version of the six-block example described in the previous section. CPU execution times shown in Table 2 indicate that simple modifications to the inversion algorithm can produce sizable increments in computer efficiency, thereby, making it feasible to invert large data sets into large reservoir models.

Multi-block test case. A relatively more complex test case was designed to further assess the spatial resolution properties of arrays of in-situ permanent sensors of pressure and DC electrical voltage. This test case consisted of 15 permeability and porosity block segments, and 6 resistivity block segments. We considered the same well and sensor deployment described in the previous case. Permanent pressure data were simulated assuming the same flow-rate pulsing schedule used in the six-block test case. Multi-pulse pressure data recorded between 100 and 300hrs of the injection schedule were input to the inversion together with DC electrical voltages. Figures 11, 12, and 13 describe inversion results for permeability, porosity, and resistivity, respectively, together with their actual fields. These results indicate a very good reconstruction of the actual permeability and resistivity fields. The effects of nonlinear coupling of flow within the multi-block formation appear to have more adverse affect on the inversion of the porosity field compared to the inversion of other petrophysical parameters. In this case, only, features of the model located in close proximity to the injection well can be accurately reconstructed.

Undoubtedly, permeable formations to be encountered in practical applications will be more spatially and petrophysically complex than the idealized models studied in this paper. Despite the spatial complexity of the underlying petrophysical model, the inversion results described above indicate that the joint use of pressure and resistivity data does help to reduce non-uniqueness in the inference of petrophysical properties. The inversion tools developed in this paper could also be used to optimize the deployment of sensors to optimally detect and quantify dynamic variations in hydrocarbon reservoir conditions.

Conclusions

Our inversion results can be regarded as a proof-of-concept exercise to appraise the spatial resolution properties of pressure and DC resistivity data acquired with in-situ permanent sensors. For all of the cases investigated in this paper, the joint inversion of pressure and electric data improved the accuracy of estimations performed separately with either data set. This was possible because simultaneous use of the two data sets naturally reduces non-uniqueness and hence improves stability. However, coupling of the two data sets is not trivial. Ideally, a rigorous multi-phase fluid flow formulation should be used to drive the simulation and inversion of the two data sets. The inversion exercises presented in this paper were based on an approximate formulation of multi-phase flow that allowed us to couple pressure and electrical phenomena through geometrical parameters. More challenging work remains to couple the two phenomena at a deeper physical level.

Technical issues explored in this paper included: (1) spatial configurations of sensor deployment, (2) noisy and imperfect data sampling strategies, and (3) modalities for the excitation of pressure and DC resistivity data. A multitude of sensor and data configurations could be further explored to
appraise the relative influence of these three issues on the accuracy and stability of the inversions. The inversion examples studied in this paper suggest that the cooperative use of pressure and DC resistivity data does provide an efficient way to track petrophysical changes due to dynamic reservoir conditions. However, because of the underlying diffusion phenomena governing the two sets of data, an enhancement of spatial resolution persists in the region close to sources and detectors. It is envisioned that the deployment of arrays of in-situ permanent sensors should be designed to improve sensitivity and resolution to deeper spatial regions in the reservoir. Challenging work remains ahead to integrate deep sonic measurements with pressure and DC resistivity data.

Acknowledgements
We would like to express our gratitude to Baker Atlas, Halliburton, Schlumberger, and Anadarko for funding of this work through UT Austin’s Center of Excellence in Formation Evaluation. We are obligated to Vladimir Druskin of Schlumberger-Doll Research and to his former coworker at the Central Geophysical Expedition in Russia, Leonid Knizhnerman, for making their forward modeling code “NKARD” available to us to perform the DC electrical response numerical simulations in this paper.

References
Appendix A – Forward Modeling of Single-Phase Fluid Flow

Let us consider a Newtonian fluid in a rigid porous medium occupying a bounded domain \( \Omega \subset \mathbb{R}^3 \) with a smooth boundary \( \partial \Omega \). The conservation of mass over a representative control volume leads to the continuity equation\(^{26}\)

\[
-\nabla \cdot [\rho(r,t)v(r,t)] = \phi(r) \frac{\partial \rho(r,t)}{\partial t}, \tag{A-1}
\]

where \( \rho \) is the mass density, \( v \) is the velocity, \( \phi \) is the time-invariant porosity distribution, \( r \) is a point in \( \mathbb{R}^3 \) and \( t \) is time. According to Darcy’s law, the fluid velocity can be written as

\[
v(r,t) = -\frac{K}{\mu} \nabla p(r,t), \tag{A-2}
\]

where \( p \) is pressure, \( \mu \) is fluid viscosity, and \( K \) is the second-order permeability tensor. We further assume the existence of a principal coordinate system in which the permeability tensor takes on the simple diagonal form

\[
K = \begin{pmatrix} k_{xx} & k_{xy} & k_{xz} \\ k_{yx} & k_{yy} & k_{yz} \\ k_{zx} & k_{zy} & k_{zz} \end{pmatrix} = \begin{pmatrix} k_x & 0 & 0 \\ 0 & k_y & 0 \\ 0 & 0 & k_z \end{pmatrix}. \tag{A-3}
\]

Finally, for a slightly compressible fluid with constant compressibility, \( C_i \), and viscosity, \( \mu \), the pressure diffusion equation (A-1) can be written as

\[
\nabla \cdot [T(r) \cdot \nabla p(r,t)] = \phi(r) C_i \frac{\partial p(r,t)}{\partial t}, \tag{A-4}
\]

where \( T = K/\mu \) is the mobility tensor.

We solve Eq. (A-4) to numerically simulate the time-domain measurements acquired with the in-situ pressure sensors for specific flow rate (time) schedules of water injection. A pressure solution is approached using a finite-difference formulation in cylindrical coordinates via the Extended Krylov Subspace Method (EKSM)\(^{13,19}\).

Appendix B – Forward Modeling of DC Electrical Responses

With the usual constitutive relationships, and assuming a harmonic time dependence of the form \( e^{i\omega t} \), the forward operator for electromagnetic problems, defined on a finite or infinite domain \( D \), is specified by the equations

\[
\nabla \times E(r) = -i \omega \mu H(r) + \mathbf{M}_S, \tag{B-1}
\]

\[
\nabla \times H(r) = [\sigma(r) + i \omega \varepsilon(r)] E(r) + \mathbf{J}_S(r), \tag{B-2}
\]

where \( \omega \) is angular frequency, \( \mu \) is magnetic permeability, \( \varepsilon \) is dielectric permittivity, \( \sigma \) is electrical conductivity (reciprocal of resistivity), \( E \) and \( H \) are the electric and magnetic field strengths due to the impressed electric and magnetic current densities \( \mathbf{J}_S \) and \( \mathbf{M}_S \), and \( r \) is the observation point. Equations (B-1) and (B-2) are solved subject to the boundary conditions applied on \( \partial D \) of the form

\[
\alpha (\hat{n} \times \mathbf{U}) + \beta (\hat{n} \times \mathbf{U} \times \nabla \times \mathbf{U}) = S, \tag{B-3}
\]

where \( \alpha \) and \( \beta \) are constants, \( S \) is a surface magnetic or electric current, and \( \mathbf{U} \) can be either \( \mathbf{E} \) or \( \mathbf{H} \), and \( \hat{n} \) is the normal vector.

The governing equation for DC resistivity is obtained by letting \( \omega = 0 \) in (B-1) and (B-2). Setting \( \mathbf{M}_S = 0 \) yields \( \nabla \times \mathbf{E}(r) = 0 \) from which it follows that \( \mathbf{E}(r) = -\nabla \Omega(r) \) where \( \Omega \) is the electric scalar potential. Taking the divergence of the \( \mathbf{H} \) field equation yields

\[
\nabla \cdot (\sigma(r) \nabla \Omega(r)) = \nabla \cdot \mathbf{J}_S(r). \tag{B-4}
\]

We may use (B-4) directly or if an electric current of strength \( I \) is enforced at \( r_S \) then \( \nabla \cdot \mathbf{J}_S(r) = -I \delta(r-r_S) \) and (B-4) simplifies to

\[
\nabla \cdot (\sigma(r) \nabla \Omega(r)) = -I \delta(r-r_S). \tag{B-5}
\]

In this paper, a solution of Eq. (B-4) is approached by using a simulation algorithm based on the semi-discrete scheme described by Druskin and Knizhnerman\(^{20}\).

Appendix C – Derivation of the Basic Equations for Gauss-Newton Inversion Algorithm

We undertake the estimation (inversion) of \( \mathbf{m} \) from the measured data by minimizing the quadratic cost function, \( C(\mathbf{m}) \), defined by Torres-Verdín and Habashy\(^{10}\), namely,

\[
2C(\mathbf{m}) = \left[ \| \mathbf{W}_d \cdot ( \mathbf{d}(\mathbf{m}) - \mathbf{d}_{\text{obs}} ) \|^2 + \chi^2 \right] + \lambda \| \mathbf{W}_m \cdot ( \mathbf{m} - \mathbf{m}_R ) \|^2. \tag{C-1}
\]

Various parameters involved in (C-1) are described in the main body of the paper.

To obtain a stationary point, \( \mathbf{m} \), at which the quadratic cost function attains a minimum, we employ a Gauss-Newton fixed-point iteration scheme\(^{25}\). Let \( \mathbf{m}^k \) be a pivoting vector where \( k \) is the iteration count. The first three terms of the Taylor series expansion of \( C(\mathbf{m}) \) about the pivoting point \( \mathbf{m}^k \) are given by

\[
C(\mathbf{m}) = C(\mathbf{m}^k) + \nabla^T C(\mathbf{m}^k) \cdot (\mathbf{m} - \mathbf{m}^k) + \frac{1}{2} (\mathbf{m} - \mathbf{m}^k)^T \nabla^2 C(\mathbf{m}^k) \cdot (\mathbf{m} - \mathbf{m}^k) \tag{C-2}
\]
\[
V = \left[ \frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \ldots, \frac{\partial}{\partial x_j}, \ldots, \frac{\partial}{\partial x_N} \right]^T,
\]
and the matrix \( VV C(m) \) denotes the Hessian of the cost function. It then follows that,
\[
VC(m) = J^T(m) \cdot W_d^T \cdot W_d \cdot [d(m) - d_{\text{obs}} ]
+ \lambda W_x^T \cdot W_x \cdot (m - m_R),
\]
where \( J(m) \) is the Jacobian matrix of \( C(m) \), written as
\[
J(m) = \left[ \begin{array}{ccc}
\frac{\partial d_1}{\partial m_1} & \ldots & \frac{\partial d_1}{\partial m_j} & \ldots & \frac{\partial d_1}{\partial m_N} \\
\vdots & & \ddots & & \vdots \\
\frac{\partial d_M}{\partial m_1} & \ldots & \frac{\partial d_M}{\partial m_j} & \ldots & \frac{\partial d_M}{\partial m_N}
\end{array} \right]_{M \times N},
\]
\[
(C-3)
\]
We implemented a first-order forward difference (Fréchet derivative) formula to approximate the entries of the Jacobian matrix\(^{12} \), i.e.,
\[
\frac{\partial d_j}{\partial m_l} = d_j(m_l + \Delta m_l) - d_j(m_l) \over \Delta m_l,
\]
where \( \Delta m_l \) was chosen as small as possible without compromising the intrinsic accuracy of the forward modeling codes (approximately 1%). The entries of the Jacobian matrix are subsequently updated as the values of \( m_l \) change and the iterations progress in their way toward the extremum of the cost function. As implied in (C-6), this approach requires two forward modeling runs per unknown. In order to accelerate the computation of the Jacobian matrix, we experimented with the fact that the Jacobian matrix as a whole could be updated using the values taken by the same matrix from previous iterations. To achieve this, we use the Broyden’s rank-one update formula\(^{26} \), i.e.,
\[
J(m^{k+1}) = J(m^k) + [\Delta d(m^k) - J(m^k) \cdot \Delta m^k] \cdot \frac{[\Delta m^k]^T}{[\Delta m^k]^T \cdot [\Delta m^k]},
\]
(C-7)
where
\[
\Delta d(m^k) = d(m^{k+1}) - d(m^k) \quad \text{and} \quad \Delta (m^k) = m^{k+1} - m^k.
\]
Here, we assume that \( d(m^k) \) changes in linear fashion with respect to \( m^k \) along the step direction \( \Delta m^k \). Often, this remains a good approximation provided that \( m^k \) is close to the extremum of the least-squares cost function. Subsequent to the computation of the Jacobian matrix using Fréchet derivatives, we make use of Broyden’s rank-one update formula through a few iterations. If a significant convergence toward the minimum is not observed, we proceed to reset the Jacobian matrix by performing the numerical simulations required to computing its entries. Such a procedure proved to be highly efficient in solving the inversion problems described in this paper.

In the Gauss-Newton iterative scheme, one discards the second-order derivatives of the cost function about the pivot point \( m^k \). Consequently, the approximate representation of the Hessian becomes
\[
VV C(m) \equiv J^T(m) \cdot W_d^T \cdot W_d \cdot J(m) + \lambda W_m^T \cdot W_m.
\]
By reordering (C-2) we obtain
\[
\Phi(m - m^k) = C(m) - C(m^k)
= V^T C(m^k) \cdot (m - m^k)
+ 1 \over 2 (m - m^k)^T \cdot \nabla V C(m^k) \cdot (m - m^k),
\]
(C-10)
where \( \Phi \) is a quadratic function. This latter function exhibits a stationary point at \( (m - m^k) \) if the gradient vector of \( \Phi \) vanishes at \( (m - m^k) \). We set
\[
\nabla \Phi(m - m^k) = V V C(m^k) \cdot (m - m^k) + C(m^k) = 0,
\]
(C-11)
whereupon
\[
V C(m^k) = J^T(m^k) \cdot W_d^T \cdot W_d \cdot [d(m^k) - d_{\text{obs}} ]
+ \lambda W_x^T \cdot W_x \cdot (m^k - m_R),
\]
(C-12)
and
\[
V V C(m^k) = J^T(m^k) \cdot W_d^T \cdot W_d \cdot J(m^k) + \lambda W_m^T \cdot W_m.
\]
(C-13)
By substituting (C-12) and (C-13) into (C-11) and by rearranging terms, we obtain the final form to compute the next iteration point. At a given level of iteration, \( m = m^{k+1} \), one has
\[
m^{k+1} = \{ J^T(m^k) \cdot W_d^T \cdot W_d \cdot J(m^k) + \lambda W_m^T \cdot W_m \}^{-1}
\cdot \{ J^T(m^k) \cdot W_d^T \cdot W_d \cdot [d(m^k) - d_{\text{obs}} + J(m^k) \cdot m^k ]
+ \lambda W_m^T \cdot W_m \cdot m_R \},
\]
(C-14)
subject to the physical bounds
\[
l_i \leq m_i^{k+1} \leq u_i.
\]
(C-15)
TABLE 1- PETROPHYSICAL AND FLUID-FLOW PARAMETERS

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water viscosity, $\mu_w$</td>
<td>1.0 cp</td>
</tr>
<tr>
<td>Total compressibility, $C_t$</td>
<td>$2.0 \times 10^{-5}$ psi$^{-1}$</td>
</tr>
<tr>
<td>Wellbore radius, $r_w$</td>
<td>0.06895 m</td>
</tr>
<tr>
<td>Reservoir external radius, $r_e$</td>
<td>107.1120 m</td>
</tr>
<tr>
<td>Reservoir thickness, $h$</td>
<td>6.1 m</td>
</tr>
<tr>
<td>Injection rate, $q$ (Step-function pulse)</td>
<td>200 rbbl/d</td>
</tr>
</tbody>
</table>

TABLE 2- COMPARISON OF CPU EXECUTION TIMES FOR CONVENTIONAL AND DUAL-GRID INVERSION TECHNIQUES

Noisefree porosity-resistivity joint Inversion: 15 model parameters

<table>
<thead>
<tr>
<th>Grid I Size (outer loop)</th>
<th>Grid II Size (inner loop)</th>
<th>CPU time (seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jacobian computed from Fréchet derivatives</td>
<td>Jacobian computed using Broyden’s rank-one update formula</td>
<td>$10^5 \times 304$ - 105 nodes 8566.704</td>
</tr>
<tr>
<td>Jacobian computed from Fréchet derivatives</td>
<td>Jacobian computed using Broyden’s rank-one update formula</td>
<td>$10^5 \times 304$ - 105 nodes 6448.892</td>
</tr>
<tr>
<td>Jacobian computed from Fréchet derivatives</td>
<td>Jacobian computed using Broyden’s rank-one update formula</td>
<td>$54 \times 204$ - 54 nodes 4486.782</td>
</tr>
</tbody>
</table>

Fig. 1- Graphical description of a generic permanent sensor installation. In-situ pressure gauges and point-contact DC electrodes are deployed in direct hydraulic contact with the formation. In this example, water is injected through an open interval thereby displacing in-situ oil. Water invasion fronts in the form of cylinders are used to indicate variability in the vertical distribution of permeability (or porosity) and resistivity.

Fig. 2- Three-layer, nine-block reservoir model. (a) Description of the reservoir model. Formation layers are in pressure communication. The permeable medium is subjected to a 135hr-long constant rate injection pulse. Fall-off pressure response is computed for the subsequent 100hr. The associated reservoir and fluid properties are listed in Table 1 except that the step function flow rate used in this validation exercise is equal to 500rbbl/d. (b) Comparison of EKSMS and ECLIPSE 100 pressure fall-off solutions in the time-domain. Solutions are computed at points vertically centered within each layer and in close radial proximity to the well boundary ($r = 0.1$m). (c) Comparison of solutions of EKSMS and ECLIPSE 100 at a second set of locations farther into the reservoir ($r = 1.05$m).
Fig. 3- Six-block formation model. (a) Description of the permeable medium. Formation layers are in pressure communication. The associated reservoir and fluid properties are listed in Table 1. Injection flow rates are modeled by way of a truncated-line source equivalent to a fully penetrated well. Both injection and observation wells are equipped with in-situ permanent pressure sensors. A resistivity array consisting of point-contact electrodes is deployed along the injection well. (b) Finite difference grid \((134 \times 249)\) used for the forward and inverse modeling of pressure data in this paper. (c) Superimposed plots of pressure change and flow rate as a function of time. The excitation history is made up of 100hr-long injection and 50hr-long fall-off periods. For the multi-pulse inversion approach we used the pressure data measured during first fall-off, second injection, and second fall-off periods. Pressure data acquired from both sensor assemblies deployed in the injection and observation wells are input to the inversion. (d) Multi-pulse pressure data measured by the sensors deployed along the injection well. (e) Multi-pulse pressure data measured by the sensors deployed along the observation well. For single-pulse inversion application, we only used the pressure data measured during the first fall-off period.
Fig. 4- (a) Finite-difference radial grid constructed with 201 logarithmically distributed subsections. This mesh is used for the forward and inverse modeling of DC resistivity data (voltages) in this paper. The forward modeling algorithm used in this paper makes use of a semi-discrete numerical simulation method that only requires a radial grid. (b) DC electrical response: voltage data measured by point-contact electrodes along the injection well.

Fig. 5- Six-block formation example. Actual and post-inversion permeability fields. Inversion results obtained using noise-free pressure and resistivity data.

Fig. 6- Six-block formation example. Actual and post-inversion porosity fields. Inversion results were obtained using noise-free pressure and resistivity data.
SPE 77621

NUMERICAL SENSITIVITY STUDIES FOR THE JOINT INVERSION OF PRESSURE AND DC RESISTIVITY MEASUREMENTS ACQUIRED WITH IN-SITU PERMANENT SENSORS

Fig. 7- Six-block formation example. Actual and post-inversion resistivity fields. Inversion results were obtained using noise-free pressure and resistivity data.

Fig. 8- Six-block formation example. Example of the fit in data space of injection well pressures. The inversion of porosity and resistivity was performed jointly from pressure and resistivity data (multi-pulse pressure data was input to the inversion).

Fig. 9- Six-block formation example. (a) Actual and (b), (c) post-inversion porosity fields. Gaussian, zero-mean, 1% random noise was added to the synthetic measurements. The post-inversion porosity field was obtained from the inversion of (b) single-pulse and (c) multi-pulse pressure data jointly with DC resistivity data (voltages).
Fig. 10- Six-block formation example. (a) Actual and (b), (c) post-inversion permeability fields. Gaussian, zero-mean, 1% random noise was added to the synthetic measurements. The post-inversion permeability field was obtained from the inversion of (b) single-pulse and (c) multi-pulse pressure data jointly with DC resistivity data (voltages).

Fig. 11- Multi-block formation example. (a) Actual and (b) post-inversion permeability fields. The post-inversion permeability field was obtained from the inversion of noise-free multi-pulse pressure data jointly with DC resistivity data (voltages).
Fig. 12- Multi-block formation example. (a) Actual and (b) post-inversion porosity fields. The post-inversion porosity field was obtained from the inversion of noise-free multi-pulse pressure data jointly with DC resistivity data (voltages).

Fig. 13- Multi-block formation example. (a) Actual and (b) post-inversion resistivity fields. Post-inversion resistivity field was obtained from the inversion of DC resistivity data (voltages measured by the permanently installed electrode array) jointly with noise-free multi-pulse pressure data.