A high-order generalized extended Born approximation to simulate electromagnetic geophysical measurements in inhomogeneous and anisotropic media

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Summary

Extensive computer resources are needed to accurately solve large-scale electromagnetic (EM) problems in inhomogeneous and anisotropic media. In this paper, we introduce a generalized extended Born approximation and its high-order variants to efficiently and accurately simulate EM geophysical problems. A generalized series expansion for the internal electric field is used to construct high-order terms of the generalized extended Born approximation (Ho-GEBA). The Ho-GEBA approximation is efficiently solved using fast Fourier transforms, with its low-order terms as efficient to compute as the first Born approximation and the extended Born approximation (EBA). A salient feature of the Ho-GEBA is its enhanced accuracy over Born and EBA, even when one considers only the first-order term of the approximation. A unique feature of Ho-GEBA is that it can be used to simulate the EM response of electrically anisotropic media. This feature is not possible with an approximation based on the use of background electric fields. Numerical examples drawn from the field of geophysical induction logging are used to benchmark the applicability, efficiency, and accuracy of the Ho-GEBA. These examples consider a tri-axial array-induction instrument and 3D dipping and anisotropic rock formations subject to invasion. Simulations performed with the Ho-GEBA are superior in accuracy and efficiency to those of Born and EBA. The accuracy of the simulations is as good as 90% in most cases.

Introduction

Approximation strategies are frequently used to solve EM scattering problems. They represent a good compromise between computer efficiency and accuracy when solving large-scale inverse scattering problems. Several approximations have been proposed and used in the past. These include Born (1933), Extended Born (EBA) (Habashy *et al.*, 1993; and Torres-Verdín and Habashy, 1994), and Quasi-Linear (Zhadnov and Fang, 1996). In addition, Gao *et al.*, 2003a, Gao *et al.*, 2003b, and Gao et al., 2004 developed a smooth approximation that makes use of field decomposition to simulate the EM response of electrically anisotropic media. The Born approximation is restricted to low frequency and low conductivity contrasts (Habashy *et al.*, 1993). On the other hand, the Extended Born approximation is superior to the Born approximation because of the inclusion of multiple scattering (Habashy et al., 1993). It has been found, however, that the EBA does not perform well when the scatterer is very close to the source region, or when the electric field exhibits significant spatial variations within the scatterer (Torres-Verdín and Habashy, 1994, and Gao *et al.*, 2003). These situations frequently occur in applications of geophysical induction logging (Gao and Torres-Verdín, 2004). Moreover, both the Born approximation and the EBA cannot properly account for EM scattering due to electrically anisotropic media. The latter situation has been analyzed in great detail in several of our previous publications (Gao *et al.*, 2003a, Gao *et al.*, 2003b, and Gao *et al.*, 2004).

Recently, we developed an efficient high-order generalized extended Born approximation (Ho-GEBA) that provides accurate simulations irrespective of the source position and the spatial distribution of the internal electric field (Gao and Torres-Verdín, 2004a, Gao and Torres-Verdín, 2004b, and Gao and Torres-Verdín, 2004c). The low-order term of the Ho-GEBA is as efficient to compute as the Born and EBA, but is substantially more accurate. We have used the Ho-GEBA to simulate 3D scattering problems (Gao and Torres-Verdín, 2004a), 2.5D borehole induction problems (Gao and Torres-Verdín, 2004b), and 3D anisotropy problems (Gao and Torres-Verdín, 2004c). This paper summarizes the theory behind the Ho-GEBA, and illustrates its applicability by solving 2.5D and 3D borehole induction problems involving conductivity anisotropy in the context of geophysical induction logging.

Theory

Assume an EM source that exhibits a time harmonic dependence of the form e^{-iwt}, where $i=\sqrt{-1}$, w is the angular frequency, and t is time. Also assume a scatterer or electrical anomaly embedded in an unbounded homogeneous and isotropic background of complex conductivity equal to s_b. The wave equation for the electric and magnetic fields, **E** and **H**, respectively, can be cast into two integral equations, given by

$$\mathbf{E}=\mathbf{E}_b+G_t^e\left(\Delta\overline{\overline{s}}\cdot\mathbf{E}\right), \qquad (1)$$

and

$$\mathbf{H}=\mathbf{H}_b+G_t^h\left(\Delta\overline{\overline{s}}\cdot\mathbf{E}\right)\cdot \qquad (2)$$

In the above equation, $\mathbf{E}_b$ and $\mathbf{H}_b$ are the electric and magnetic field vectors, respectively, associated with the background medium and the impressed sources. The variables G_t^e and G_t^h are linear integral operators, defined by

$$G_t^e(\cdot) = \int_t \overline{\overline{G}}^e(\mathbf{r},\mathbf{r_0})(\cdot)d\mathbf{r}_0 , \qquad (3)$$

and

$$G_t^h(\cdot) = \int_t \overline{\overline{G}}^h(\mathbf{r},\mathbf{r_0})(\cdot)d\mathbf{r}_0 , \qquad (4)$$

respectively, where $\overline{\overline{G}}^e(\mathbf{r},\mathbf{r}_0)$ and $\overline{\overline{G}}^h(\mathbf{r},\mathbf{r}_0)$ are the electric and magnetic dyadic Green's function, respectively, the subscript t represents the spatial support of the operator, and $\Delta\overline{s}$ is the material complex conductivity anomaly with respect to the background medium.

In practice, equation (1) is solved via the method of moments. This requires substantial computer resources, including memory storage and CPU time, because of the need to construct and solve a dense matrix equation.

<u>Theory of the Generalized Extended Born Approximation</u>

Suppose that M is the total number of spatial discretization cells. We decompose the domain t into two sub-domains, t_s and $t-t_s$, in which t_s is a sub-domain which encloses the m-th cell. Thus, equation (1) can be rewritten in component form as

$$\mathbf{E}_m - G_{t_s}^e\left(\Delta\overline{s}\cdot\mathbf{E}\right) = \mathbf{E}_{bm} + G_{t-t_s}^e\left(\Delta\overline{s}\cdot\mathbf{E}\right), \text{ m=1, 2,…, M}. \qquad (5)$$

Theorem 1: If there exists a t_s which satisfies the following two conditions:

1) Condition 1: Within t_s, the electric field $\mathbf{E}$ can be treated as spatially invariant, and
2) Condition 2: Outside of t_s, the Green's dyad decreases in amplitude sufficiently fast to become negligible,

then the second term on the right-hand side of equation (5) could be neglected without affecting the accuracy of the result.

According to Theorem 1, for such a sub-domain t_s, one obtains the basic equation of the Generalized Extended Born Approximation (GEBA)

$$\mathbf{E}_m = \overline{\overline{\Lambda}}_m \cdot \mathbf{E}_{bm} , \qquad (6)$$

where $\overline{\Lambda}_m$ is a scattering tensor for the m-th cell, given by

$$\overline{\overline{\Lambda}}_m = \left(\overline{\overline{\mathrm{I}}} - G_{t_s}^e\left(\Delta\overline{\overline{s}}\right)\right)^{-1} . \qquad (7)$$

Two special cases can be derived for the GEBA:

Special Case 1: When $t_s \to t_m$, where t_m is the singular domain, which only encloses the m-th cell. This results in the simplest scattering tensor, namely,

$$\overline{\overline{\Lambda}}_m^{(s1)} = \left(\overline{\overline{\mathrm{I}}} - G_{t_m}\left(\Delta\overline{\overline{s}}\right)\right)^{-1} . \qquad (8)$$

However, the above expression may not be sufficiently accurate since this treatment violates Condition 2 in Theorem 1, i.e. the Green's dyad may not fall off sufficiently fast to cause the second term on the right-hand side of equation (5) to be negligible.

Special Case 2: When $t_s \to t$, one has the most complex form of scattering tensor, namely,

$$\overline{\Lambda}_m^{(s2)} = \left(\overline{\mathrm{I}} - G_t^e\left(\Delta\overline{s}\right)\right)^{-1} . \qquad (9)$$

This result is identical to that of the Extended Born Approximation (EBA) (Habashy et al., 1993, and Torres-Verdín and Habashy, 1994). Computation of the scattering tensor given by equation (9) requires computer resources proportional to $O\left(M^2\right)$. Also, equation (9) may not provide accurate simulations since it violates Condition 1 in Theorem 1 (the electric field, in general, may not be spatially invariant in the whole domain).

<u>Theory of the high-order GEBA (Ho-GEBA)</u>

In the previous section, we assume a sub-domain t_s which satisfies Theorem 1. However, we remark that the two conditions in Theorem 1 are in opposite directions. Thus, the existence of t_s is a trade-off between meeting Condition 1 and Condition 2. In general, a t_s which simultaneously satisfies Condition 1 and Condition 2 may not exist. An alternative strategy is to choose a t_s, which satisfies Condition 1 as closely as possible, and to approximate the electric field $\mathbf{E}$ on the right-hand side of equation (5) in some fashion. We proceed to show how to develop such a strategy using the generalized series (GS) for the internal electric field (Gao and Torres-Verdín, 2004a).

According to Gao and Torres-Verdín (2004a), GS is expressed as

$$\mathbf{E}(\mathbf{r}) = \sum_{n=0}^{\infty} \mathbf{E}_{CB}^{(n)}(\mathbf{r}) , \qquad (10)$$

where

$$\mathbf{E}_{CB}^{(n)} = \overline{\overline{a}} \cdot G_t^e\left(\Delta\overline{\overline{s}} \cdot \mathbf{E}_{CB}^{(n-1)}\right) + \overline{\overline{b}} \cdot \mathbf{E}_{CB}^{(n-1)}, \text{ n>1}, \qquad (11)$$

$$\mathbf{E}_{CB}^{(1)} = \overline{\overline{a}} \cdot G_t^e\left(\Delta\overline{\overline{s}} \cdot \mathbf{E}_{CB}^{(0)}\right) + \overline{\overline{a}} \cdot \left(\mathbf{E}_b - \mathbf{E}_{CB}^{(0)}\right), \qquad (12)$$

$$\bar{\bar{a}} = 2s'_b\left(2s'_b\bar{\bar{I}} + \Delta\bar{\bar{s}}\right)^{-1}, \quad (13)$$

$$\bar{\bar{b}} = \bar{\bar{I}} - \bar{\bar{a}}, \quad (14)$$

and s'_b is the real part of s_b. In the above equations, $E^{(0)}_{CB}$ is an arbitrary starting point, depending on whether one desires to construct the classical Born series (Born 1933), the modified Born series (Zhadnov and Fang, 1997), or the quasi-linear series (Zhadnov and Fang, 1997).

If a sub-domain t_s can be found that satisfies Condition 1 completely and Condition 2 in some approximate way, one can substitute GS into equation (5) to obtain the basic equation of Ho-EBA, i.e.,

$$\mathbf{E}_m(\mathbf{r}) \approx \sum_{n=0}^{N-1} \mathbf{E}^{(n)}_{CBm}(\mathbf{r}) + \bar{\bar{\Lambda}}_m(\mathbf{r}) \cdot \mathbf{E}'^{(N)}_{CBm}(\mathbf{r}), \quad (15)$$

where $\mathbf{E}'^{(N)}_{CBm}$ is given by

$$\mathbf{E}'^{(N)}_{CBm} = G^e_t\left(\Delta\bar{\bar{s}} \cdot \mathbf{E}^{(N-1)}_{CBm}\right) + \bar{\bar{g}} \cdot \mathbf{E}^{(N-1)}_{CBm}, \text{ N>1}, \quad (16)$$

$$\mathbf{E}'^{(1)}_{CBm} = G^e_t\left(\Delta\bar{\bar{s}} \cdot \mathbf{E}^{(0)}_{CBm}\right) + \mathbf{E}_{bm} - \mathbf{E}^{(0)}_{CBm}, \quad (17)$$

and

$$\bar{\bar{g}} = \frac{\Delta\bar{\bar{s}}}{2s'_b}. \quad (18)$$

A salient feature of Ho-EBA is that it is a combination of GS and GEBA, with GEBA acting as the residual term of GS. However, numerical exercises show that the GEBA term can dramatically increase the convergence of the GS, thereby rendering the HO-GEBA extremely efficient to accurately solve EM scattering problems. If an optimal sub-domain t_s can be constructed, then the GEBA may result in accurate solutions. However, as has been pointed out by Gao *et al*. (2003a), Gao *et al*. (2003b), and Gao *et al*. (2004), because of null components in the background field vector $\mathbf{E}_b$, the GEBA may not properly reproduce cross-coupling EM terms in the presence of electrically anisotropic media. This problem can be circumvented with the HO-GEBA.

Two special cases can also be derived for the Ho-GEBA:

Special Case 1: Substitution of $\bar{\bar{\Lambda}}_m$ in equation (15) for $\bar{\bar{\Lambda}}^{s1}_m$ yields

$$\mathbf{E}_m(\mathbf{r}) \approx \sum_{n=0}^{N-1} \mathbf{E}^{(n)}_{CBm}(\mathbf{r}) + \bar{\bar{\Lambda}}^{s1}_m(\mathbf{r}) \cdot \mathbf{E}'^{(N)}_{CBm}(\mathbf{r}) \cdot \quad (19)$$

This form of HO-GEBA closely follows the assumptions made in the derivation of the HO-GEBA; hence, it could be a very good approximation to solve EM scattering problems. Since the scattering tensor may be far from optimal, equation (19) may converge slower than equation (15) with an optimal scattering tensor.

Special Case 2: Substitution of $\bar{\bar{\Lambda}}_m$ in equation (15) for $\bar{\bar{\Lambda}}^{s2}_m$ yields an approximation corresponding to the special Case 2 of the GEBA, namely,

$$\mathbf{E}_m(\mathbf{r}) \approx \sum_{n=0}^{N-1} \mathbf{E}^{(n)}_{CBm}(\mathbf{r}) + \bar{\bar{\Lambda}}^{s2}_m(\mathbf{r}) \cdot \mathbf{E}'^{(N)}_{CBm}(\mathbf{r}) \cdot \quad (20)$$

We remark, however, that equation (20) cannot be derived directly from equation (5), because when $t_s \to t$, the term involving $t - t_s$ in equation (5) tends to zero, and only the term $\mathbf{E}_{bm}$ remains.

All of the above described approximations, including the GS, the GEBA, and the Ho-GEBA, have been described in detail by Gao and Torres-Verdín (2004a, 2004b, and 2004c).

Numerical Examples

For the numerical examples shown below, we consider only the special case of the Ho-GEBA (up to the 2nd order term of the expansion) given by equation (19).

Figure 1 describes an axisymmetric formation model and an array induction-type instrument. The corresponding formation and tool parameters are shown on the same figure. We assume that the measurement is the difference of the magnetic fields sensed by the two magnetic receivers. Because of space limitations, Figure 2 shows only the comparison between the convergence of the GS and Ho-GEBA expansions. The plot clearly shows that the Ho-GEBA expansion converges much faster than the GS expansion.

Figure 3 shows a five-layered 3D formation model (where the second and fourth layers exhibit conductivity anisotropy) and a tri-axial EM induction instrument. The formation is referred to as "1D" without borehole and mud-filtrate invasion; otherwise a "3D" model is used in the simulations. The dip angle is assumed to be 60^o. Simulation results shown in Figure 4 consist of the H_{xx}, H_{yy}, and H_{zz} measurements simulated for the 1D formation (left-hand panel) and for the 3D formation (right-hand panel). These results clearly show the enhanced accuracy of the Ho-GEBA over the Born and EBA approximations. The acronyms 1D and 3DFDM on Figure 3 refer to an analytic 1D code, and a 3D finite difference code, respectively.

Conclusions

A high-order generalized extended Born approximation (Ho-GEBA) is proposed in this paper to simulate EM scattering problems in the presence of inhomogeneous and

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electrically anisotropic media. Theoretical analysis and numerical exercises consistently indicate that the Ho-GEBA provides more accurate simulation solutions than Born and EBA without sacrifice of computer efficiency. Moreover, the Ho-GEBA has the unique advantage over alternative approximations to properly account for EM scattering and cross-coupling due to electrical anisotropy.

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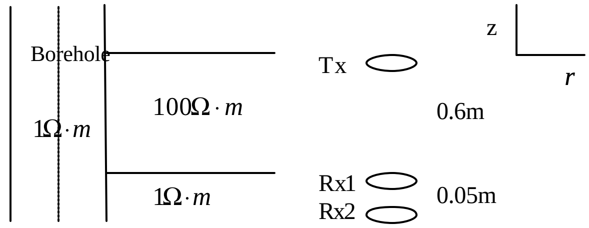

Figure 1: A 3-layered axisymmetric formation model and an array induction instrument.

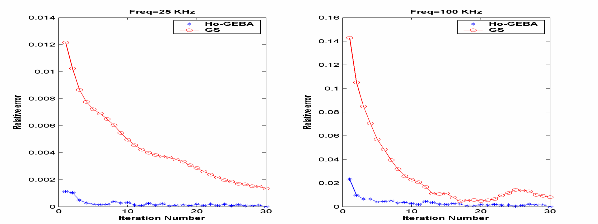

Figure 2: Comparison of the convergence of the Ho-GEBA and GS expansions for the formation model shown on Figure 1

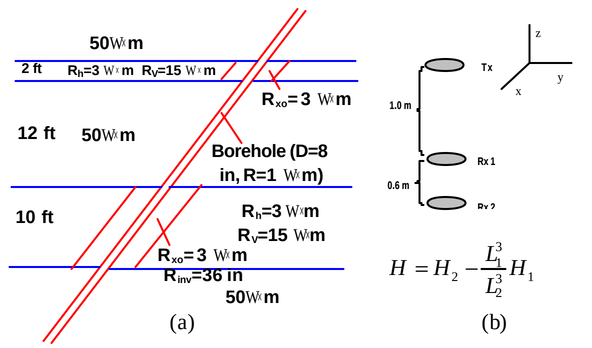

Figure 3: a) A 5-layered formation model (not to scale); b) The configuration of a tri-axial induction instrument, where the transmitters and receivers can be oriented in the x, y, and z directions. The formula indicated how the instrument measurement is synthesized.

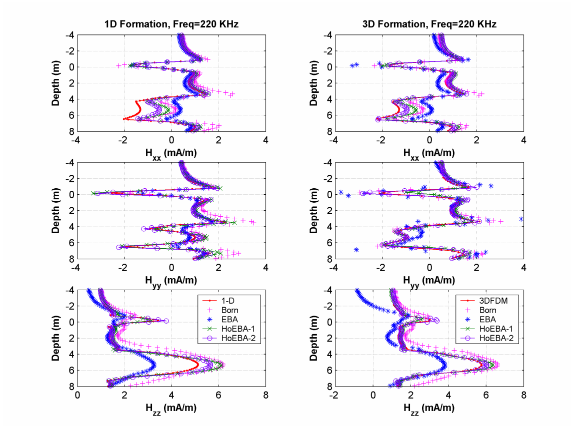

Figure 4: Simulation results for the formation model shown in Figure 3.