Z-99 Enforcing smoothness and assessing uncertainty in one-dimensional pre-stack seismic inversion

Varela, O. J., Torres-Verdín, C., Sen, M. K.

Petroleum and Geosystems Engineering, and Institute for Geophysics, The University of Texas at Austin

Abstract

Estimation of elastic properties of rock formations from surface seismic data is a subject of great interest in the exploration and development of hydrocarbon reservoirs. In this paper, a global inversion technique is used to estimate and appraise one-dimensional (1D) distributions of S-wave velocity, P-wave velocity, and density (ρ), from pre-stack surface seismic data. The objective is twofold: (a) to evaluate the effect of the choice of objective function, and of the degree and type of smoothness criterion applied to the inversion; and (b) to introduce a new stochastic inversion algorithm that efficiently combines sampling and smoothness concepts borrowed from the field of geostatistical estimation. It is shown that the choice of stochastic technique used to perform the global inversion can significantly constrain the efficiency of the estimation process. Very fast simulating annealing was found to be the most efficient global inversion technique. It is also shown that an appropriate choice of objective function is necessary for a robust and efficient match of noisy and sparse surface seismic measurements. Because of the inherent non-uniqueness of the inverse problem, provisions are necessary to control the degree and kind of smoothness criterion enforced in the estimation process. Several procedures are discussed to simultaneously enforce smoothness and assess uncertainty of the estimated elastic parameters.

Introduction

Pre-stack seismic data are often used to provide a link between petrophysical properties and surface seismic measurements. Independent elastic parameters derived from pre-stack surface seismic data include S-wave velocity (v_s), P-wave velocity (v_p), and density (ρ) as a function of depth. The problem of estimating 1D distributions of elastic parameters from pre-stack seismic data can be approached with nonlinear inversion. This procedure is equivalent to the optimization of an objective function written as the metric of the difference between the measured and numerically simulated pre-stack surface seismic data. Inversion methods based on local optimization often fail to produce a global minimum when the starting model is far from the optimal model and the objective function is multimodal (Tarantola, 1987). On the other hand, an exhaustive trial-and-error search scheme in model space cannot be implemented in an efficient manner because the model space is often extremely large (Sen and Stoffa, 1991). The efficiency of global optimization methods remains highly controlled by both the efficiency of the search algorithm and the computer power available to simulate the measured data. This paper implements a global optimization technique that is based on simulating annealing (SA, Ingber, 1989).

The numerical study presented in this paper evaluates various SA algorithms used for global optimization. A detailed study is also performed of the selection of objective function and of the degree and type of smoothness criterion enforced by the inversion. Finally, a new efficient algorithm is introduced to stochastically invert one-dimensional pre-stack seismic data that makes use of sampling strategies and smoothness concepts borrowed from the field of geostatistical estimation.

Pre-stack stochastic inversion

Background. The physical process of reflection, transmission, and mode conversion of elastic plane waves at a boundary has been described by Aki and Richards (2002). Inversion of pre-stack seismic data yields a 1D distribution of elastic parameters from the information content available in both time and source-receiver space. The inversion algorithm considered in this paper makes uses of the reflectivity method (Kennett, 1983) to calculate seismic responses for all source-receiver offsets assuming an angle-independent source wavelet.

Formulation. Inversion of pre-stack seismic data into a 1D distribution of elastic parameters remains a highly nonlinear and non-unique process. In this paper, we make use a global optimization method that implements an efficient guided search based on SA. Moreover, the estimation of model parameters is controlled by physical constrains (e.g., trends) that consistently eliminate the search of unreasonable models that also fit the data.

Global Optimization. SA is a global optimization method that stochastically simulates the cooling of a multi-particle physical system. Metropolis's procedure mimics the natural physical process whereby crystal lattices of glass or metal relax to a state of lower thermal equilibrium. This process is usually referred to as annealing (Metropolis et. al., 1953). Various SA algorithms are considered in this paper, namely, Metropolis (M), Heat Bath (HB), and very fast simulating annealing (VFSA). All three SA algorithms considered in this paper make use of the same acceptance/rejection criterion introduced by Metropolis.

Objective Function. In order to assess the match between synthetic data [$S(x,t)^{est}$] and measured data [$S(x,t)^{data}$], different types of objective functions were considered in the inversion algorithm. The first type of objective function was constructed using the L_1 and L_2 norms of the data misfit, and the second type was constructed using the geometric and harmonic metrics of data misfit described by Sen and Stoffa (1991). Equation 1 defines the harmonic objective function in the frequency domain. In this equation, x is position, f is frequency, N_{off} is the number of source-receiver offsets, N_t is the number of time samples in each trace for a given offset, and N_f is the number of frequencies in each trace for a given offset. In general, additional terms in the objective function are necessary to control the smoothness of the model parameters. These terms can be defined as the flatness and the roughness (e.g., first and second derivative) of the 1D distribution of model parameters.

$$\text{Harmonic norm: } \left\| e_f \right\|_h = \sum_1^{Noff} \left[\frac{2\sum_1^{Nf} S(x,f)^{data} S^*(x,f)^{est}}{\left(\sum_1^{Nf} S(x,f)^{data} S^*(x,f)^{data} \right)^{1/2} + \left(\sum_1^{Nf} S(x,f)^{est} S^*(x,f)^{est} \right)^{1/2}} \right] \qquad (1)$$

Proposed Inversion Algorithm. Based on extensive numerical experiments, the proposed algorithm makes use of the most efficient annealing technique (i.e., VFSA) and the most robust objective function (i.e., harmonic), and introduces a sampling strategy based on geostatistical concepts. At the outset, several hard-points are randomly chosen. Sampling of the next point in model space is performed using a semivariogram function provided that this point remains within the assumed correlation range; otherwise the point is determined using VFSA. A Metropolis acceptance/rejection criterion is enforced by the algorithm. The solution is conditioned to the starting point, whereupon various seeds are necessary to evaluate the final solution. This also provides a natural way to assess non-uniqueness and hence to quantitatively appraise the inversion results in the presence of noisy and sparsely sampled pre-stack seismic data.

Numerical Experiments and Discussion

Example 1: Annealing Technique. In this example, an inversion was performed assuming a 50-layer model. Layers have different thickness and the data are sampled at a constant rate of 2 ms in the interval from 0 to 1.5 seconds. A zero-phase Ricker wavelet centered at 35 Hz was assumed in the simulation of the pre-stack seismic data. The inversion yields estimates of v_p, v_s, and ρ, using various annealing techniques (M, HB, and VFSA) implemented under the same conditions. Figure 1 describes the CPU time associated with each algorithm in relation to the CPU time associated with VFSA. Inversion exercises considered in this paper consistently indicate that VFSA remains the most efficient SA algorithm for global optimization.

Example 2: Choice of Objective Function and Smoothness Criterion. A similar inversion exercise was performed to evaluate the effect of the choice of objective function. Figure 2 shows the original seismic data (Panel a) and the residuals for the cases of L_1 (Panel b), L_2 (Panel c), and harmonic (Panel d) objective functions. The harmonic objective function is sensitive to the absolute amplitude differences and produces superior results in terms of misfit and computational performance. Figures 3 and 4 show results when different types and degree of smoothness are enforced by the inversion. Results substantially improve when both model flatness and model roughness are included in the inversion algorithm.

Example 3: Proposed Inversion Algorithm. Figure 5 shows results in data and model space obtained using the proposed inversion algorithm. Input and inverted data, and data residuals are shown in Panel (a). A better correlation is obtained between input and inverted values of model parameters when compared to previous inversions (e.g., see example 2) as seen in Panel (b) of this figure.

Summary

Various simulating annealing algorithms and objective functions have been evaluated to perform 1D global inversion of pre-stack seismic data. Both the selection of a specific annealing technique and the construction of the objective function significantly constrain the efficiency of the inversion and the accuracy of the results. This paper introduces a new global inversion algorithm that enforces sampling and smoothness strategies using geostatistical concepts. The new inversion algorithm is also capable of rendering estimates of uncertainty in an efficient manner.

Acknowledgements

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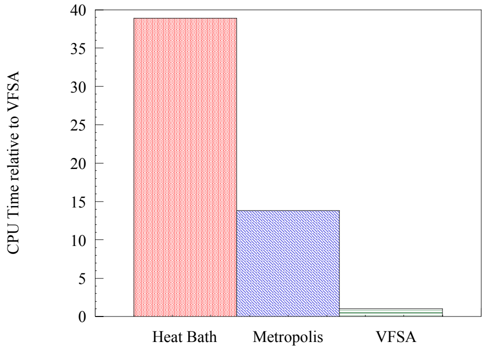

Figure 1. CPU time relative to VFSA associated with different annealing algorithms when solving the same pre-stack inversion problem.

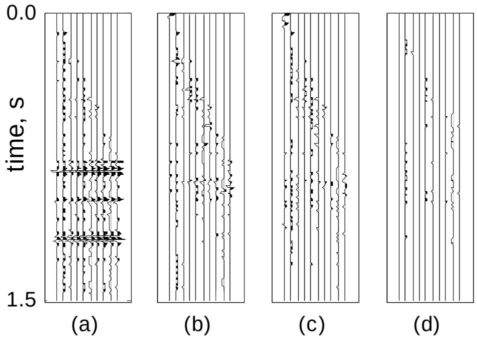

Figure 2. Input pre-stack seismic data (a), and data residuals obtained using several objective functions, namely: (b) L_1-norm, (c) L_2-norm, and (d) harmonic.

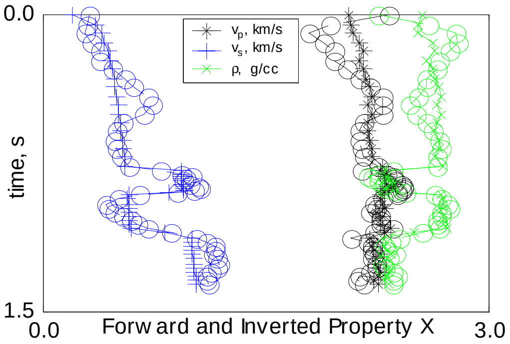

Figure 3. Input and inverted model parameters when the objective function exhibits both flatness and roughness.

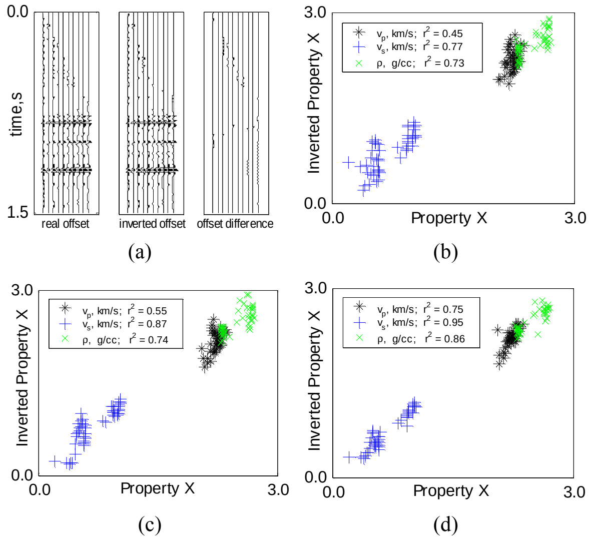

Figure 4. Inversion results obtained for various types of smoothness criterion using the same similarity in data space. (a) Input and inverted data, and data residuals. (b) Inversion results obtained without enforcing smoothness. (c) Flatness is included in the objective function. (d) Both flatness and roughness are included in the objective function.

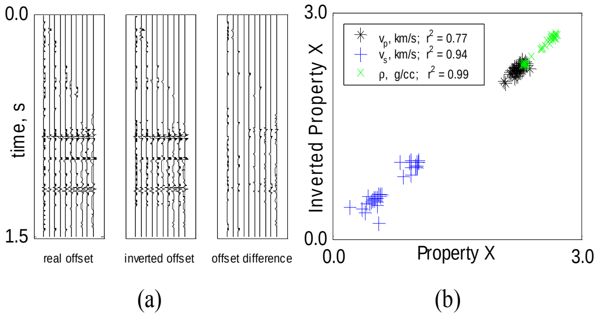

Figure 5. Results obtained with the proposed inversion algorithm: (a) Inversion results in data space, and (b) inversion results in model space.