

A data-adaptive spatial resolution method for three-dimensional inversion of triaxial borehole electromagnetic measurements

Faruk O. Alpak*, Robert K. Mallan, Junsheng Hou, and Carlos Torres-Verdín, The University of Texas at Austin

SUMMARY

We develop a novel adaptive inversion technique: Data-adaptive Spatial Resolution Inversion (DSRI) method. DSRI eliminates the need to select parameterization prior to inversion. Instead, one performs a hierarchical search for the correct parameterization while solving a sequence of inverse problems with an increasing dimension of parameterization. A parsimonious approach to the inverse problem involves the application of the same refinement applied all over the spatial domain. Such an approach may lead to over-parameterization, subsequently, to unrealistic conductivity estimates and excessive computational work. With DSRI technique, the new parameterization at an arbitrary stage of inversion sequence is allocated such that new degrees of freedom are not necessarily introduced all over the spatial domain of the electromagnetic problem. The aim is to allocate new degrees of freedom only where it is warranted by the data. Inversion results confirm that DSRI constitutes a robust technique for efficient multiparameter inversion of multicomponent electromagnetic measurements.

INTRODUCTION

Accurate and reliable determination of hydrocarbon saturation is of principal importance for decisions regarding the exploration, development, and production of thinly-laminated sand-shale sequences typically encountered in deepwater turbidite reservoirs (Zhang et al., 2004). Thinly laminated sand-shale sequences are characterized by electrical anisotropy, which in turn, could be interpreted as a good indicator of hydrocarbon pay (Klein et al., 1995). In the case of horizontally layered formations with conductivity anisotropy, the value of conductivity parallel to the bedding plane differs from the one that is perpendicular to the bedding plane. Media of the above-described conductivity characteristics are also referred to as transversely anisotropic (TI or uniaxially anisotropic) conductive media. Often, high conductivity shale laminae dominate horizontal conductivity information embodied by the magnetic field measurements while information about vertical conductivity is predominantly determined by the low conductivity hydrocarbon-bearing sand laminae. In vertical boreholes, vertical dipole antennas of conventional induction logging tools detect signal from eddy currents that flow parallel to bedding plane. As such, they lack sensitivity to vertical conductivity, thereby causing underestimation of hydrocarbon reserves. This problem was identified as early as in 1930's by Schlumberger et al. (1934) and emphasized in terms of the paradox of anisotropy by Kunz and Moran (1958). Conversely, in highly-deviated and horizontal boreholes, conventional induction logging tools are more sensitive to the commonly encountered lower vertical conductivity. This behavior introduces difficulties in identifying marker beds for well-to-well correlation and for geosteering.

Modern multicomponent induction logging tools have been introduced by Baker-Hughes (Kriegshäuser et al. 2000) and Schlumberger (Rosthal et al. 2003) to address the problems of conventional induction logging measurements in anisotropic rock formations. Baker-Hughes' tool measures five magnetic field components: H_{xz} , H_{yy} , H_{zz} , H_{xy} , and H_{xz} . On the other hand, Schlumberger's tool acquires measurements of all nine components of the magnetic field allowing closed-form determination of the azimuthal angle via a rotation of the

tensor field measurements without resorting to inversion (Wang et al., 2003).

With the exception of Abubakar et al. (2004), development of inversion algorithms for triaxial induction logging measurements has been focused to predominantly one- or, to a lesser extent, to two-dimensional models (Cheryauka and Zhdanov, 2001; Kriegshäuser et al., 2001; Lu and Alumbaugh, 2001; Yu et al., 2001; Zhang et al., 2001; Zhang and Mezzatesta, 2001; Anderson et al., 2002; Tompkins and Alumbaugh, 2002; Wang et al., 2003; Zhang et al., 2004).

A central problem when attempting to recover the spatial distribution of conductivity from electromagnetic measurements is to explicitly honor the intrinsic spatial resolution of the measurements, i.e., the *a priori* parameterization of conductivity. Too low resolution will result in measurements not being reconciled, while too high resolution leads to unnecessary computational effort due to over-parameterization. In turn, over-parameterization, when not accurately regularized, often leads to increased non-uniqueness in the inversion and ultimately causes spatial distributions of conductivity to be inconsistent with the physics of the measurements. The objective of this paper is to develop a data-adaptive spatial resolution inversion methodology for the problem of multicomponent induction logging. We seek to circumvent the difficulty of estimating the correct spatial resolution by gradually increasing the resolution only at spatial locations warranted by the measurements. Thus, instead of proposing a fixed rigid resolution and subsequently seeking to invert the corresponding parameter set all at once, the technique we formulate solves a sequence of inversion problems with monotonically increasing resolution. Similar inversion approaches have been developed for automatic history matching of production measurements in the reservoir engineering literature (see, for example, Ben Ameer et al., 2002; Grimstad et al. 2003).

DATA-ADAPTIVE SPATIAL RESOLUTION INVERSION (DSRI) METHOD

Conventional pixel-by-pixel inversion techniques attempt to compensate the resolution discrepancy in the spatial domain due to over-parameterization by penalizing deviations from *a-priori* knowledge about the sought solution. Bayesian estimation or Tikhonov regularization techniques (Tarantola, 1987; Tikhonov, 1977), for example, require the use of regularization parameters for this purpose. In many cases, the determination of regularization parameters is based on highly subjective grounds. DSRI, on the other hand, can be viewed as a sequence of inversions where the parameterization for each level of estimation is a zonation. The resolution of the zonation is increased for each step in the sequence, i.e., progressive parameterization. The process is initiated by estimating a single parameter, namely, the average conductivity for the entire medium, using an arbitrary initial value. Then, the medium is split into 2^n equally sized subregions, where n is the spatial dimension of the conductive medium. A parameter is estimated for each subregion with initial values equal to the estimated parameter value for the entire conductive medium. In an approach that operates with first-order complexity following the first-step spatial splitting, each of these subregions can be divided into 2^n equal-size new subregions, and the refinement proceeds recursively with the powers of 2 until the convergence criteria are met to satisfaction. Let us

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denote the number of complexity levels by m . Then, the number of parameters to be estimated in an arbitrary instance of inversion is given by $K = 2^{n(m-1)}$. It is self-evident that K will increase rapidly as the sequential refinement process advances, especially, for problems formulated in two- or three-dimensional spatial domains. In order to keep the growth ratio for the number of parameters from one complexity level to the next under control while maintaining conformance to measurements, only a few new parameters are introduced at each level of the DSRI. Refinement indicators are developed to identify the focus region at each DSRI level. New degrees of freedom are introduced only within the confines of this region. The principal philosophy of DSRI relies on the goal of identifying the focus-region and adapting its resolution to the measurement data at each iteration level while avoiding unnecessary refinements. In turn, DSRI refinements are solely guided by objective criteria that stem from the information content of the measurements. In consequence, DSRI eliminates subjective decisions that are artifacts of the rigid over-parameterization strategy of conventional pixel-by-pixel or parametric simultaneous inversion techniques. The use of a regularization parameter, described above, constitutes a major example. DSRI eliminates the need for artificial regularization parameters by adjusting the parameter resolution on the spatial domain based on the information warranted by the measurements.

A variant of the DSRI technique is developed to address parametric inverse problems. For such problems, the inversion process starts with a simple parametric model. The initial model is constructed using a set of model parameters with the most impact on the reduction of the objective function. Identification of this set of model parameters is achieved at the first assessment level also called the first complexity assessment-advancement level. Once the level of model complexity is determined, the inversion phase starts (complexity level inversion phase). Nonlinear inversion iterations are advanced until the improvement in the inverted model becomes insignificant or the reduction of the objective function indicates that complexity level convergence is attained. This, in turn, triggers the next complexity assessment-advancement level of the adaptive inversion. Assessment-advancement level involves the identification of the most relevant set new model parameters and their introduction to the inversion process. Subsequent to each assessment-advancement level DSRI reverts back to the inversion phase. Data-adaptive inversion process advances through a number of complexity assessment-advancement and complexity level inversion phases until the complexity assessment-advancement phase determines that the next advancement will be statistically insignificant. Since the model complexity is continuously adjusted to the information content of the measurements, DSRI algorithm by nature avoids the use of artificial regularization parameters. Consequently, DSRI eliminates uncertainty introduced by the subjective choice of artificial regularization parameters. This is by no means a claim that the uncertainty in the inverted model is completely eliminated. In fact, uncertainty is still prevalent, yet, the uncertainty in the inverted model stems purely from imperfect observations (measurements) collected at finite locations and from finite angles within the unknown model. Elimination of subjective regularization parameters renders the DSRI a powerful tool for uncertainty analysis. At the inversion phase, DSRI solver minimizes the following objective function:

$$C(\mathbf{x}) = [\mathbf{m} - S(\mathbf{x})]^T \mathbf{C}_D^{-1} [\mathbf{m} - S(\mathbf{x})], \quad (1)$$

subject to physical upper and lower bounds on the vector of model parameters, \mathbf{x} . In equation (1), \mathbf{m} denotes the vector of multicomponent electromagnetic measurements and \mathbf{C}_D represents the covariance matrix of measurement errors. The vector of simulated measurements, on the other hand, is given by $S(\mathbf{x})$. In principle, any optimization method can be utilized to compute model parameters by minimizing the objective function of equation (1) at each level of DSRI complexity. Optimization techniques that require the gradient of the objective function are computationally less intense in terms of computational effort per iteration. Techniques that require the computation of a sensitivity matrix are computationally more demanding compared to gradient-based optimization techniques but in generally entail more rapid convergence. In practice, the optimal choice of optimizer is problem dependent. DSRI focus-region tracking logic makes use of sensitivity matrix to sort through various parameterizations and select the most suitable ones to proceed further. In this paper, we implement a sensitivity-matrix-based optimization method to naturally complement the DSRI technique. DSRI optimization engine is implemented using a least-squares Gauss-Newton optimization algorithm (Nocedal and Wright, 1999) embedded in a dual finite-difference grid stencil (Torres-Verdín et al., 2000).

FORWARD MODEL

Robust and accurate simulation of triaxial induction logging measurements is accomplished using an efficient three-dimensional anisotropic forward model developed using a coupled potential formulation. Details of this highly efficient forward model are described in Hou and Torres-Verdín (2003; 2004; 2005). In addition to these developments, for the purpose of robust and efficient parametric inversion, an anisotropic conductivity averaging scheme is implemented following the guidelines of Wang and Fang (2001). Computational efficiency of the forward modeling algorithm is further enhanced using an optimal geometric finite-difference gridding technique (Druskin et al., 1999).

NUMERICAL EXAMPLE

In a numerical example, we consider inversion of synthetic multicomponent electromagnetic measurements generated from a transversely anisotropic invaded single-layer model. The layer is assumed to be buried in an isotropic background with a conductivity of 0.1 S/m. In addition to invasion, we also consider the presence of a conductive annulus between the invaded and virgin zones. Conductivity values are chosen in consistency with the physics of fresh water-base mud-filtrate invasion into a formation saturated with high-salinity residual aqueous and movable hydrocarbon phases (see, for more details, George et al., 2004). The true model parameters are displayed in Table 1. Triaxial induction logging measurements are simulated for a single-frequency (20 kHz) single-transmitter induction logging tool with six multicomponent receivers. Measurements are acquired in an inclined borehole with a dip angle of 60°, and simulated for 40 logging stations across the -20 ft to +20 ft (true relative vertical depth) interval. In total, $40 \times 6 \times 5$ measurements are used for inversion. A parametric variant of the DSRI is applied to solve the inverse problem. The relative progress of adaptive refinements performed by DSRI are shown in the panels of Figure 4. Level-by-level inversion results are displayed in Table 1. Post-inversion data fit is shown in Figure 5. In this latter figure, synthetic measurements and post-inversion simulated electromagnetic data are displayed for an example receiver location.

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CONCLUSIONS

A data-adaptive inversion technique is formulated and implemented for the estimation of spatial distributions of conductivity from borehole electromagnetic field measurements by operating on fairly coarse scales. For cases where the true conductivity distribution contains fine-scale variations, DSRI is guided to introduce new degrees of freedom as complex as warranted by the measurements thereby eliminating subjective regularization decisions that introduce artificial biases into the inversion process. The algorithm relies on progressive spatial domain refinement in conjunction with sequential parameter estimation. Focus-region identification and tracking criteria are utilized to prevent over-parameterization. DSRI algorithm by nature avoids the use of artificial regularization parameters. Inversion results validate the robustness and computational efficiency of DSRI.

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Table 1: Summary of inversion results for the single invaded layer example. Values of parameters that are subject to inversion at a given DSRI level are typed with boldface characters along with values of parameters describing the true conductivity model.

MODEL PARAMETER	MODEL UNIT	INITIAL MODEL	L1 MODEL	L2 MODEL	L3 MODEL	L4 MODEL	L5 MODEL	TRUE MODEL
θ	[degree]	55.00	59.36	59.39	59.95	59.69	60.03	60.00
σ_{ht}	[S/m]	0.100	0.011	0.014	0.015	0.012	0.013	0.013
α_{ht}	[dimensionless]	1.00	1.00	1.95	2.16	2.47	2.48	2.50
z_0	[ft]	-4.08	-4.08	-4.08	-4.02	-4.05	-4.02	-4.00
Δz	[ft]	8.16	8.16	8.16	8.00	8.06	8.00	8.00
σ_{htw}	[S/m]	0.100	0.100	0.100	0.100	0.052	0.012	0.010
α_{htw}	[dimensionless]	1.00	1.00	1.00	1.00	1.56	1.48	1.50
r_{inv}	[in]	15.00	15.00	15.00	15.00	8.68	10.13	10.00
σ_{hms}	[S/m]	0.100	0.100	0.100	0.100	0.100	0.023	0.025
α_{hms}	[dimensionless]	1.00	1.00	1.00	1.00	1.00	1.41	1.50
r_{ms}	[in]	25.00	25.00	25.00	25.00	25.00	19.50	20.00
No of inverted parameters:		-	2	3	5	8	11	-

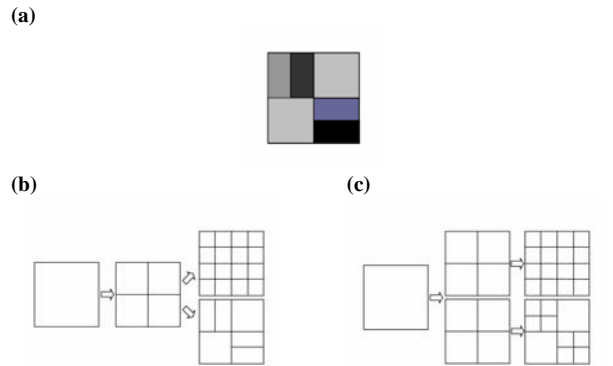


Figure 1: (a) A simple two-dimensional inverse problem. (b) Over-parameterization due to regular hierarchical refinement strategy versus ideal parameterization for the inverse problem of interest. (c) Regular hierarchical refinement strategy versus adaptive hierarchical refinement strategy.

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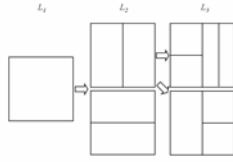


Figure 2: Levels of refinement for the adaptive hierarchical refinement strategy. Three levels are shown in this figure. At the second level, vertical refinement is selected over horizontal refinement.

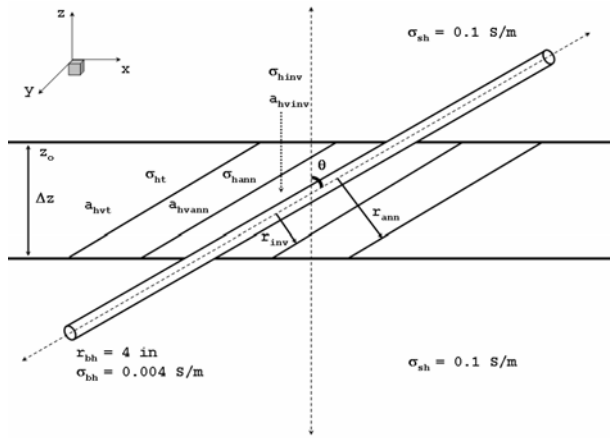


Figure 3: Example single-layer model for the parametric inversion of multicomponent induction logging measurements.

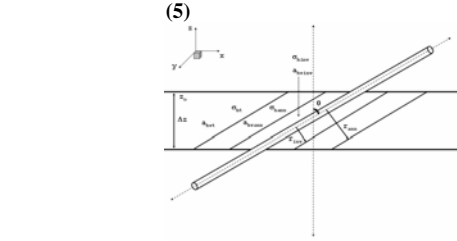
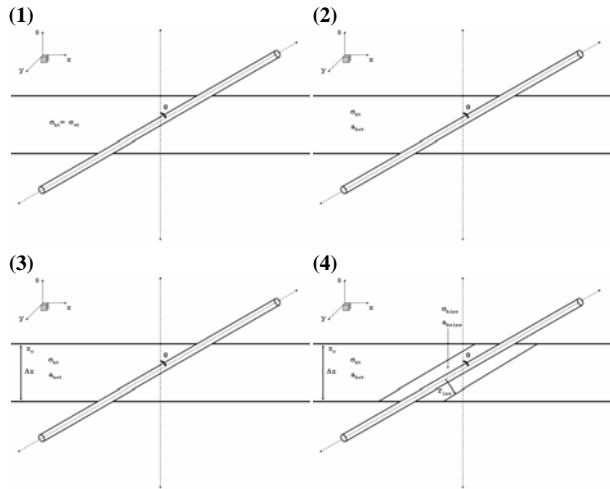


Figure 4: Five phases of the adaptive hierarchical parametric refinement strategy. Multicomponent induction logging inverse problem: single-layer example.

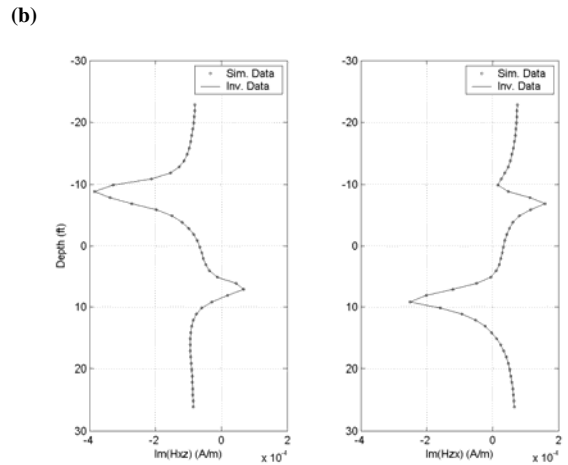
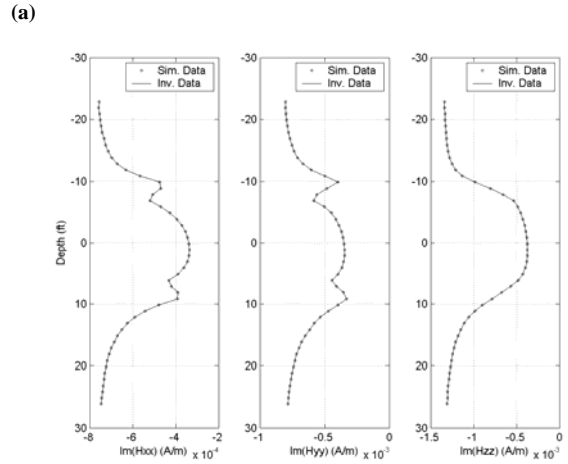


Figure 5: Post-inversion data fit for an example receiver location.